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NATURALNESS RE-EXAMINED: IMPLICATIONS FOR SUPERSYMMETRY SEARCHES

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We discuss the origin of "focus points" in the scalar mass RGEs of the MSSM and their implications for collider searches. We present a new exact analytic solution to the homogeneous system of scalar mass RGEs in the MSSM for general $\tan\beta$. This is then used to prove that the focus point for $m_{H_u}^2$ depends only on the value of the top Yukawa coupling at the *weak* scale (not its value at the GUT scale) and is independent of the bottom Yukawa coupling.

1 Focus Points in Supersymmetry

Supersymmetry (SUSY) is the leading candidate for new physics beyond the Standard Model. It provides a framework for naturally explaining the stability of the weak scale with respect to radiative corrections. In its minimal version, the Minimal Supersymmetric Standard Model (MSSM), it also successfully predicts gauge coupling unification. Both of these attractive features of SUSY hinge on the assumption that the scale of the superpartner masses is not too far above the electroweak scale. This has reinforced a widespread optimism that the next round of collider experiments at the Tevatron, LHC or the NLC are guaranteed to discover all superpartners, if they exists.

In SUSY, the weak scale is determined by the relevant model parameters – the soft SUSY breaking Higgs masses, m_{H_u} and m_{H_d} , and the supersymmetric Higgs mass parameter, μ :

$$\frac{1}{2}m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \sim -m_{H_u}^2 - \mu^2, \quad (1)$$

where the last relation holds for large $\tan\beta$. Naturalness requires that there be no large cancellations in the RHS of eq. (1) when expressed in terms of the *fundamental* parameters $\{a_i\}$ of the model (e.g., in minimal supergravity (mSUGRA), $\{a_i\} = \{m_0, M_{1/2}, \mu_0, A_0, B_0\}$). Otherwise an unwanted hierarchy is reintroduced, and the appeal of SUSY as a solution to the hierarchy problem is lost. The degree of fine-tuning involved in eq. (1) is usually quantified in terms of the sensitivity coefficients¹ $c_{a_i} \equiv |\partial \ln m_Z^2 / \partial \ln a_i|$.

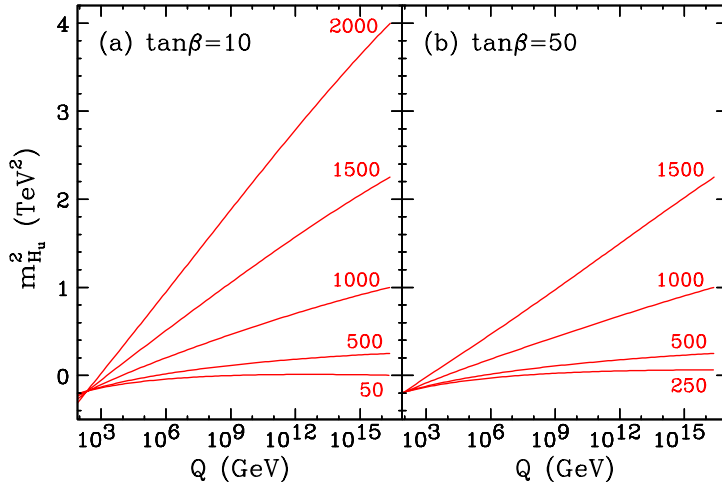


Figure 1: The RG evolution of $m_{H_u}^2$ for (a) $\tan\beta = 10$ and (b) $\tan\beta = 50$, several values of m_0 (shown, in GeV), $M_{1/2} = 300$ GeV, $A_0 = 0$, and $m_t = 174$ GeV.

Recently we pointed out² that a general class of supersymmetric theories, including mSUGRA, exhibits “focus points” in the MSSM renormalization group equations (RGEs). (See also Ref.³.) In particular, for the experimentally interesting range of top quark mass values, $m_t \sim 170 - 180$ GeV, m_{H_u} has such a focus point at the weak scale and is therefore highly insensitive to its GUT scale boundary value m_0 (see Fig. 1).

The focus point for $m_{H_u}^2$ implies that c_{m_0} is small². In Fig. 2 we show contours of the overall sensitivity parameter $c \equiv \max\{c_{a_i}\}$. We see that regions of parameter space with $m_0 \sim 2 - 3$ TeV are as natural as regions with $m_0 \lesssim 1$ TeV. Hence, it is quite possible that all squarks and sleptons have multi-TeV masses, making their discovery at colliders significantly more challenging than conventionally expected. On the other hand, Fig. 2 reveals that naturalness restricts the gaugino mass $M_{1/2}$ to the few hundred GeV range.

The focus point mechanism is surprisingly robust against variations of the input parameters, e.g., $M_{1/2}, A_0, m_t$ and M_{GUT} . We will prove analytically^a that the focus point scale is also independent of $\tan\beta$. This supplements previous numerical demonstrations of this result². In the process (see section 2) we shall derive analytic solutions to the homogeneous parts of the scalar mass RGEs for arbitrary values of $\tan\beta$. To our knowledge, such solutions

^aWe are grateful to Jon Bagger for stimulating questions and discussions.

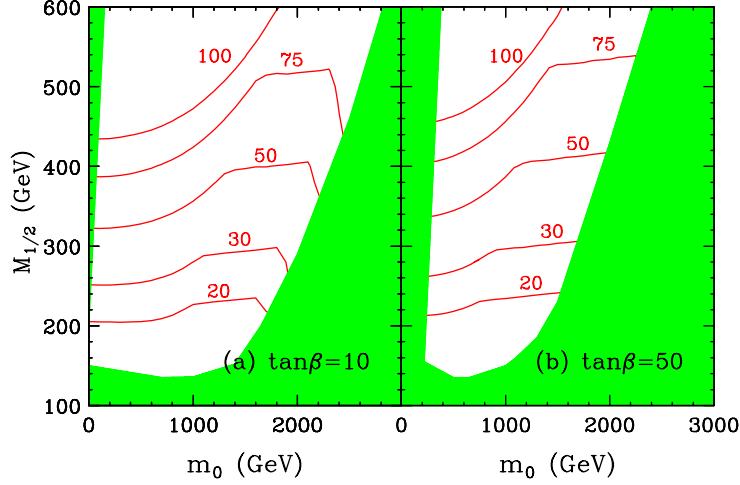


Figure 2: Contours of constant sensitivity parameter c in the $(m_0, M_{1/2})$ plane for (a) $\tan \beta = 10$ and (b) $\tan \beta = 50$, $A_0 = 0$, $\mu > 0$, and $m_t = 174$ GeV. The bottom and right shaded region is excluded by the chargino mass limit of 90 GeV. The top left region is also excluded if a neutral LSP is required.

have not been presented previously in the literature.

2 Analytic Solution to the Homogeneous System of Scalar RGEs

Neglecting the tau Yukawa coupling and the hypercharge differences for top and bottom, the RGEs for the relevant Yukawa couplings read

$$\dot{Y}_t = Y_t (6Y_t + Y_b - a_i g_i^2), \quad \dot{Y}_b = Y_b (6Y_b + Y_t - a_i g_i^2), \quad (2)$$

where $Y_t \equiv y_t^2$, $Y_b \equiv y_b^2$, $8\pi^2 t \equiv \ln Q$ and $\dot{} \equiv d/dt$. The homogeneous system of RGEs for the relevant scalar masses is

$$\dot{\vec{M}} = \mathbf{N} \vec{M}, \quad (3)$$

where $\vec{M}^T = (m_{H_u}^2, m_U^2, m_Q^2, m_D^2, m_{H_d}^2)$ and

$$\mathbf{N} = Y_t \begin{pmatrix} 3 & 3 & 3 & 0 & 0 \\ 2 & 2 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + Y_b \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \end{pmatrix}. \quad (4)$$

Now perform a suitable change of variables

$$\begin{pmatrix} m_{H_u}^2 \\ m_{U_3}^2 \\ m_{Q_3}^2 \\ m_{D_3}^2 \\ m_{H_d}^2 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 0 & 0 & 0 \\ -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 3 & -1 \end{pmatrix} \begin{pmatrix} c_0 \\ c_t(t) \\ c'_0 \\ c_b(t) \\ c''_0 \end{pmatrix}, \quad (5)$$

which amounts simply to decomposing \vec{M} into the eigenvectors of \mathbf{N} (those are just the columns of the 5×5 matrix in the RHS of eq. (5)). Since \mathbf{N} has three zero eigenvalues, three of the new variables, namely c_0 , c'_0 and c''_0 , are constants and do not run. We have thus reduced the system of five equations (3) to a system of two equations for the remaining new variables c_t and c_b :

$$\dot{c}_t = Y_t (6c_t + c_b), \quad \dot{c}_b = Y_b (6c_b + c_t). \quad (6)$$

The GUT scale boundary conditions (BC) are

$$c_t(t_G) = k_t, \quad c_b(t_G) = k_b, \quad (7)$$

where k_t and k_b are some model-specific pure numbers. In the absence of a general method for solving the system (6), we make the following ansatz:

$$c_t(t) = k_t + Y_t(t)f_t(t), \quad c_b(t) = k_b + Y_b(t)f_b(t), \quad (8)$$

with $f_t(t_G) = f_b(t_G) = 0$. Then, we get

$$\dot{f}_t = 6k_t + k_b + a_i g_i^2 f_t - Y_b(f_t - f_b), \quad (9)$$

and \dot{f}_b is given by replacing the subscript $t \leftrightarrow b$. Now, by calculating $\dot{f}_t - \dot{f}_b$, we find a simple equation for $f_t - f_b$,

$$(\dot{f}_t - \dot{f}_b) + (Y_t + Y_b - a_i g_i^2)(f_t - f_b) = 5(k_t - k_b), \quad (10)$$

which can be easily integrated. To simplify notation, let us define

$$Y(t) \equiv \int_{t_G}^t dt' (Y_t(t') + Y_b(t')), \quad G(t) \equiv \int_{t_G}^t dt' a_i g_i^2(t'). \quad (11)$$

Then the solution to eq. (10) can be written as

$$f_t(t) - f_b(t) = 5(k_t - k_b) e^{G(t) - Y(t)} \int_{t_G}^t dt' e^{-G(t') + Y(t')}, \quad (12)$$

where we have fixed the integration constant using $k_t = k_b$. It is now trivial to solve for $f_t(t)$ and $f_b(t)$ themselves. The final solution for $c_t(t)$ is

$$c_t(t) = k_t + Y_t(t)e^{G(t)} \left\{ (6k_t + k_b) \int_{t_G}^t dt' e^{-G(t')} - 5(k_t - k_b) \int_{t_G}^t dt' Y_b(t') e^{-Y(t')} \int_{t_G}^{t'} dt'' e^{Y(t'') - G(t'')} \right\}. \quad (13)$$

The solution for $c_b(t)$ is obtained simply by interchanging $t \leftrightarrow b$ in (13). As analytic solutions for Y_t and Y_b are known⁴, this then completes the analytic solution of the homogeneous scalar mass RGEs *for arbitrary* $\tan \beta$.

Now, we discuss the $\tan \beta$ independence of the focus point scale of $m_{H_u}^2$. Define the focus point scale $Q_F^{(H_u)}$ as

$$\left. \frac{dm_{H_u}^2}{dm_0} \right|_{Q=Q_F^{(H_u)}} = 0. \quad (14)$$

The m_0 dependence of $m_{H_u}^2$ can be studied by using the homogeneous part of the RGE for the squared scalar masses, and hence $Q_F^{(H_u)}$ is obtained by solving the following equation:

$$c_0 + 3c_t(t_F) \equiv 0, \quad (15)$$

where $8\pi^2 t_F = \ln Q_F^{(H_u)}$. In mSUGRA we find $c_0 = -\frac{2}{7}m_0^2$, while $c_t(t_F)$ is given by eq. (13) with $t = t_F$ and $k_t = k_b = \frac{3}{7}m_0^2$:

$$c_t(t_F) = \left(\frac{3}{7} + 3Y_t(t_F)e^{G(t_F)} \int_{t_G}^{t_F} dt' e^{-G(t')} \right) m_0^2. \quad (16)$$

The boundary condition $k_t = k_b$ leads to an extreme simplification, as the second term in the brackets in eq. (13) vanishes. Substituting (16) into eq. (15), we obtain the focus point condition in the form

$$Y_t(t_F)e^{G(t_F)} \int_{t_F}^{t_G} dt' e^{-G(t')} = \frac{1}{9}. \quad (17)$$

The focus point may therefore be written in terms of only the gauge couplings and $y_t(t_F)$. The extremely mild $\tan \beta$ dependence of $Q_F^{(H_u)}$ for moderate to large $\tan \beta$ then follows from $y_t(t_F) \propto 1/\sin \beta \sim \text{const} + \mathcal{O}(\tan^{-2} \beta)$. Note that as $\tan \beta$ varies in this range, y_b and $y_t(t_G)$ will vary. The variation in $y_t(t_G)$ was claimed in Ref.⁵ to destroy the insensitivity of m_{H_u} to m_0 . We see, however, that this variation is irrelevant for the focus point: the focus point depends on $y_t(t_F)$, not $y_t(t_G)$, and for all moderate and large $\tan \beta$, the focus point remains at the weak scale, and the sensitivity coefficient c_{m_0} is small.

3 Implications For Supersymmetry Searches

We have seen that the naturalness bound on m_0 (i.e., the typical sfermion mass) may be as large as a few TeV (see Fig. 2). This result has important implications for the superpartner spectrum and, in particular, the discovery prospects for scalar superpartners at future colliders.

Multi-TeV sleptons (whose masses scale as $\sim m_0$), for example, are beyond the kinematic limit $\sqrt{s}/2$ of all proposed linear colliders and will also escape detection at hadron colliders, as they are not strongly produced, and will rarely be obtained in the cascade decays of strongly interacting superparticles. Multi-TeV squarks will, of course, also evade proposed linear colliders. They will also stretch the LHC reach, although a recent study⁶ in the lepton plus jets channel has revealed discovery potential even for squarks up to 3 TeV.

In contrast to the sfermions, gauginos and higgsinos cannot be very heavy in this scenario. For example, the constraint $c \leq 50$ implies $M_{1/2} \lesssim 400$ GeV, corresponding to $M_1 \lesssim 160$ GeV, $M_2 \lesssim 320$ GeV, and $M_3 \lesssim 1.2$ TeV. Such gauginos will be produced in large numbers at the LHC, and will be discovered in typical scenarios. (However, the detectability of all gauginos and higgsinos in cases of mass degeneracies at the LHC remains an open question.)

Finally, in spite of the relatively heavy squark masses allowed by the focus point mechanism, the most natural range for the light Higgs mass is somewhere below 118 GeV. This is because large values of $|A_0|$ (and hence large stop mixing) are disallowed by naturalness, as they induce too large $m_{H_u}^2$ through the RGEs. Therefore, even if all sleptons and squarks are very heavy, Run II of the Tevatron has a golden opportunity to explore much of the most natural mSUGRA parameter space in its search for the lightest Higgs boson.

Acknowledgments

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